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What Can We Theoretically Know about Uncertainity Shocks?

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ABSTRACT

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Séverine MENGUY (2022). What Can We Theoretically Know about Uncertainity Shocks? Journal of Global Economy, Trade and International Business. 2(2), 151-177. https:/ /DOI: 10.47509/ JGETIB.2022.v02i02.03 This paper studies the consequences on various economic and prices variables of an uncertainty shock on the preferences of households (demand shock) or on labor productivity (supply shock). The main results of our theoretical model are then as follows. A demand shock on the preferences of the representative consumer /household increases uncertainty about the future and precautionary saving; this implies recessionary tensions. In case of such a negative demand shock, prices (deflation), real and nominal wages decrease, whereas the markup over labor marginal costs increases. A negative supply shock on labor productivity is also recessionary on global economic activity: it implies a decrease of public expenditure and of private consumption. However, a higher volume of labor force must then be employed to compensate for its weaker productivity. Besides, the nominal and the real wage, the markups over labor and capital marginal costs all decrease, in a background of moderate inflationary tensions. Furthermore, the main effect of uncertainty per se and of the variance of shocks would be to increase the potential variance of the public debt level, which would be much more difficult to control.

Keywords: uncertainty shocks, households' preferences, labor productivity

JEL Classification Codes: E27, E32, E60

1. INTRODUCTION

In 1921, Knight already provided a famous distinction between risk, associated with probabilities, and uncertainty, the inability to forecast the likelihood of events happening. Afterwards, at least since Keynes, uncertainty about future effective demand and production costs has been considered as a fundamental determinant of investment and economic activity. Therefore, economic literature has tried to identify the roots of main business fluctuations. Real Business Cycle models consider that technology or productivity shocks are the main source of economic cycles. Other papers underline the importance of monetary or fiscal shocks. But uncertainty per se is also considered as a determinant which can drive macroeconomic fluctuations. Therefore, the potential role of an uncertain economic framework in influencing economic activity has been widely studied in the recent economic literature, in particular since the financial crisis of 2008. This question is of the highest interest, and it became all the more relevant in particular in the background of the huge world-wide COVID-19 health crisis.

In this framework, stylized facts in the economic literature often underline that macro-economic uncertainty (VIX index related to the Standard & Poor's, bond yields, exchange rates variations, mentions in newspapers articles, forecast errors of key macroeconomic variables, etc.) or micro-economic uncertainty (at the industry or plant level, wages) is often associated with recessions [see Bloom (2014)]. Nevertheless, the question remains to determine if a real relation of causality exists between uncertainty and recessions, and what is the direction of this causality. Uncertainty and recessions unambiguously appear to be correlated empirically. Therefore, it is difficult to insulate the recessionary consequences on investment and consumption due to the recession itself from those due to uncertainty of the global framework.

First, investment is more volatile in an uncertain economic framework. Indeed, physical capital can be difficult to remove once it has been realized, and the financial cost of a fixed investment is difficult to make profitable if realized opportunities are weaker than anticipated. In the same way, employment is more volatile in case of recessions, which sometimes necessitates, for workers in sectors with difficulties, recruitment and training in new sectors and technologies with more opportunities. In the same way, for households, uncertainty may delay some investment, in particular regarding real estate, and increase their precautionary sparing instead of consuming [see Bloom (2014)]. Besides, in an uncertain framework, the efficiency of monetary and fiscal policies to influence economic activity may be weaker; the interest rate sensitivity of economic activity or the budgetary multiplier are reduced, and productivity growth is also more limited. Finally, from a financial point of view, higher uncertainty must be compensated with higher risk premiums for investors, because of their risk aversion.

In the theoretical literature, Bloom (2009) offers a structural framework to analyze the impact of uncertainty shocks. He builds a model with a time-varying second moment, based on firm-level data between 1962 and 2008. He finds that a macro uncertainty shock produces a rapid drop and rebound in aggregate output and employment, because higher uncertainty causes firms to temporarily pause their investment and hiring, whereas productivity growth also falls because this pause in activity freezes reallocation across units. Besides, Fajgelbaum *et al.* (2017) develop a theory of endogenous uncertainty and business cycles in which short-lived shocks can generate long-lasting recessions. In their model, higher uncertainty about fundamentals discourages investment, and the economy displays uncertainty traps. While the economy recovers quickly after small shocks, large temporary shocks may have long-lasting effects to lower activity. However, Born and Pfeifer (2021) assess that in New-Keynesian DSGE models, the theoretical hypothesis of sticky wages (nominal rigidities) is important to generate an increase in wage markups and this negative effect of uncertainty shocks.

Empirically, Berger et al. (2020) find that innovations in realized stock market volatility are robustly followed by contractions, while shocks to forward-looking uncertainty (second-moment news shocks) and expected volatility have no significant effect on the economy. However, other papers find a non-negligible influence of an uncertain framework on global economic variables. For example, using data for the United-States and the United-Kingdom between 1979 and 2011, Leduc and Liu (2016) show that uncertainty shocks act like a negative aggregate demand shock, which raises unemployment and lowers inflation. Furthermore, Caggiano et al. (2020) mention that COVID-19 induced a peak of uncertainty and volatility, captured by the variation of a forward looking proxy like the VIX index in March 2020. In the same way, Baker and Blum (2013) use natural disasters, terrorist attacks, political coups and revolutions for sixty countries since 1970; they consider stock market returns in levels and volatility as proxies of first and second moment shocks on business conditions. Then, they find that both are highly significant, with second moment shocks accounting for at least a half of the variation in GDP growth. Furthermore, using both the traditional measure of uncertainty - stock market volatility index - and a new one -number of New York Times' articles - in the United-States between 1962 and 2008, Alexopoulos and Cohen (2009) show that uncertainty shocks can generate short sharp recessions and recoveries.

Using the Chicago Board Options Exchange Volatility Index between 1986 and 2014, Basu and Bundick (2017) show that in a sticky prices framework, an uncertainty shock about the future causes significant declines in output, consumption, investment, and hours worked. Indeed, in a recessionary framework, households don't only self-insure by reducing consumption; they also work more ('precautionary labor supply') for a given real wage, which reduces marginal production costs and increases firm markups. Therefore, global demand (investment and consumption), employment and labor demand, hours worked and the real wage decrease. Besides, higher uncertainty has even more negative effects if monetary policy can no longer perform its usual stabilizing function because of the zero lower bound. Caggiano *et al.* (2017) also find that the contractionary effects of uncertainty shocks are statistically larger when the Zero Lower Bound is binding, between 1962 and 2015 in the United States.

Furthermore, using a small-scale Structural VAR modeling for the United-States during the period 1960-2015, Ludvingson *et al.* (2015) find that sharply higher macroeconomic and policy uncertainty in recessions is often an endogenous response to output shocks, amplifying downturns caused by other shocks, while uncertainty about financial markets is a likely exogenous source of output fluctuations causing recessions. Gourio (2012) also introduces a small, time-varying risk of economic disaster in a standard real business cycle model. He finds that shocks to the perceived probability of disaster, which generate time-varying risk premiums, are equivalent to preference shocks, and can lead to a collapse of investment and a recession.

From a micro-economic point of view, Bloom et al. (2018) show that in the United-States, between 1972 and 2011, at the industry as well as at the firm or at the establishment (plant) level, the average growth rate of total factor productivity and output decreases and their variance increases during recessions. Indeed, in a recessionary framework, firms stop hiring and reduce hours worked, whereas investment also decreases. Besides, uncertainty also implies labor misallocation, as it is less directed towards the most productive firms and sectors. In the same way, Bloom et al. (2007) study a large sample of UK manufacturing companies between 1972 and 1991. They show that with (partial) irreversibility, higher uncertainty reduces the responsiveness of investment to demand shocks ('wait and see' option). Guiso and Parigi (1999) also investigate empirically the effects of uncertainty on the investment decisions of a sample of about 1000 Italian manufacturing firms in 1992-1993. They show that uncertainty weakens the response of investment to demand, thus slowing down capital accumulation. Besides, the effect of uncertainty on investment is stronger for firms that cannot easily reverse investment decisions, with substantial market power, or for those that are likely to face inelastic demand. In the same way, using a heterogeneousfirm Dynamic Stochastic General Equilibrium model, based on German data between 1973 and 1998, Bachman and Bayer (2013) show that an unexpected increase in profitability risk perceived by firms lead them to adopt a 'wait-andsee' policy for investment. However, the authors assume that time-varying firmlevel risk through 'wait-and-see' dynamics is weak and limited, and it is unlikely a major source of business cycle fluctuations. The main explanatory power of an uncertainty shock would mainly rely in its contribution to predict future productivity shocks at a longer term horizon.

Quite differently, Bachmann and Moscarini (2011) explore the hypothesis that the causation may run the opposite way. According to them, a negative first moment shock to profitability induces a more risky behavior from firms, which in turn raises observed cross-sectional dispersion and time series volatility of individual economic outcomes. So, in a context of uncertainty, firms would have an incentive to modify their strategies to survive, and prices' volatility would increase. Furthermore, with micro-data at the firm level, Gilchrist *et al.* (2010) underline the importance of financial distortions as the main mechanism through which fluctuations in uncertainty affect macroeconomic outcomes. Financial market frictions and innovations in credit spreads would have a strong effect on the user cost of capital and investment.

Therefore, empirical studies are much more numerous than analytical studies about the theoretical implications of a context characterized by more uncertainty on global economic variables; nevertheless, it is important to try to formalize what can be known theoretically about uncertainty shocks. So, the current paper provides a precise analytical analysis of the consequences of two kinds of uncertainty shocks in a simple macro-economic framework, and the rest of the paper is organized as follows. The second section describes a simple New-Keynesian model, with a representative firm and a representative household, monetary and budgetary authorities. The third section analyzes the consequences of two different uncertainty shocks: on households' preferences and on labor productivity, for global macroeconomic variables: demand factors and prices variables. Finally, the fourth section concludes the paper.

2. THE MODEL

Our model includes a representative household, a representative firm, a central bank choosing the nominal interest rate and a government choosing public expenditure in order to stabilize various shocks. We allow the possibility of sticky prices by introducing a Calvo-type framework, which is important to generate the co-movements between the decrease of economic activity and hours worked in case of an uncertainty shock observed in empirical data and stylized facts. Indeed, Basu and Bundick (2017) show that neo-classical models

with flexible prices are unable to reproduce these data. We introduce productivity (cost or supply) shocks and households' discount rate (demand) shocks, with a time-varying second moment, representing uncertainty about future demand or supply costs. Given global demand, the necessary labor demand depends on the production function; global demand defines the production level (output) and the necessary labor supply. Afterwards, this labor supply is made consistent with household optimization by having the markup taking on its required value.

The previous section has mentioned important stylized facts, which must be reproduced in the framework of our theoretical model. A negative uncertainty (higher variance) shock implies a higher risk of recession. Therefore, firms temporarily pause their investment; they stop hiring and reduce hours worked. Productivity growth is also reduced, whereas consumption and output also decrease. Indeed, as mentioned by Basu and Bundick (2017), in a recessionary framework, households don't only self-insure by reducing consumption; they also work more ('precautionary labor supply') for a given real wage, which reduces marginal production costs. As prices are not fully flexible, firm markups above production costs increase. Therefore, these higher markups reduce global demand (investment and consumption goods), employment and labor demand, effective hours worked and the real wage.

2.1. The Representative Firm

The representative firm produces a single final good with the help of two production factors: capital and labor, which respective shares in the production function are $(0 < \alpha < 1)$ and $(0 < 1 - \alpha < 1)$. Marginal products (returns to scale) are positive and diminishing, and these two factors are complement in the production function. All corporate taxes are included in the capital income tax levied on households. So, the production function has the following form:

$$Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha} \tag{1}$$

With, in period (t): (A_i): technology or productivity shock; (Y_i): real economic activity; (K): physical capital stock; (L): labor demand.

The representative firm maximizes its nominal profit, which is as follows:

$$\Pi_t = P_t Y_t - \mu_t^K R_t K_t - \mu_t^L W_t L_t \tag{2}$$

With: (P): consumer prices; (R): nominal interest rate; (W): nominal wage rate.

So, production factors are paid at their marginal products, decreased by a markup. Indeed, $(\mu_t^K > 1)$ and $(\mu_t^L > 1)$ are the markups over capital and labor marginal costs, if we suppose that firms have market power due to monopolistic competition. We suppose that these markups adjust to clear the labor market; the real wage varies to reach equilibrium whatever the level of labor productivity. So, profit maximization implies:

$$P_t \frac{\partial Y_t}{\partial K_t} = \alpha \frac{P_t Y_t}{K_t} = \mu_t^K R_t > 0$$
(3)

$$P_t \frac{\partial Y_t}{\partial L_t} = (1 - \alpha) \frac{P_t Y_t}{L_t} = \mu_t^L W_t > 0$$
(4)

In our modelling, uncertainty can first be introduced by the way of a technological shock on labor productivity (A), which variation $[a_t = \log(A)]$ is given by a random walk:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \qquad \qquad 0 \le \rho_a \le 1 \tag{5}$$

Where (ρ_{a}) is the persistence of the shock, whereas the serially uncorrelated (independent) innovation (ε_{t}^{a}) is normally distributed with mean zero and standard deviation (σ_{t}^{a2}) . We suppose that uncertainty shocks are independent, and cannot be probabilized. Therefore, we obtain the following expected value and variance for the productivity shock:

$$E_k(a_t) = \rho_a^{t-k} a_k V_k(a_t) = \sum_{j=k}^t \rho_a^{2(t-j)} \sigma_j^{a_2}$$
(6)

- $(A_k = 1)$ and $(a_k = 0)$ in a period (k) with no technological shock
- $(A_k > 1)$ and $(a_k > 0)$ if a positive technological shock increases labor productivity
- $(A_k < 1)$ and $(a_k < 0)$ if a negative technological shock decreases labor productivity

Therefore, as in Bloom *et al.* (2018), our modeling makes the hypothesis that even if the productivity shock is small in normal times, fluctuations quickly increase with the variance of innovations. According to Baker and Blum (2013), the uncertainty of economic situation increases in particular in case of political coups, revolutions or terrorist attacks, whereas natural disasters have more short-lived consequences. The recent worldwide COVID-19 health crisis can probably be classified as a recessionary situation decreasing labor productivity ($a_i < 0$).

We also suppose that the capital stock varies according to the following equation:

$$E_t(K_{t+1}) = (1 - \delta)K_t + INV_t \tag{7}$$

(*INV*): real investment in new physical capital in (*t*), (δ): depreciation rate of physical capital.

In the long run, the capital stock is supposed to be replaced at the pace of its depreciation rate (INV = δK). So, in logarithms and in variation, the capital stock adjusts as follows:

$$E_t(k_{t+1}) = (1-\delta)k_t + \left(\frac{\mathrm{INV}}{\mathrm{K}}\right)inv_t = (1-\delta)k_t + \delta inv_t \tag{8}$$

2.2. The Representative Household

The representative household provides labor and it consumes goods. In a given period (t), it maximizes an inter-temporal utility function:

$$max \sum_{k=0}^{\infty} S_{t+k} \beta^k E_t(U_{t+k})$$
⁽⁹⁾

Where: E_t is the rational expectation operator conditional on information available at date (*t*), (β) is the time discount factor. Prices variables are taken as given.

In our modelling, as in Basu and Bundick (2017), uncertainty can be introduced by the way of a demand shock on households' preferences (S). The discount rate of the future is subject to a shock whose variation $[s_i = \log(S)]$ follows a random walk:

$$s_t = \rho_s s_{t-1} + \varepsilon_t^s \qquad \qquad 0 \le \rho_s \le 1 \tag{10}$$

Where (ρ) is the persistence of the preference shock, whereas the serially uncorrelated (independent) innovation (ε_t^s) is normally distributed with mean zero and standard deviation (σ_t^{s2}) . Therefore, we obtain the following expected value and variance for the preference shock:

$$E_k(s_t) = \rho_s^{t-k} s_k V_k(s_t) = \sum_{j=k}^t \rho_s^{2(t-j)} \sigma_j^{s_2}$$
(11)

- $(S_k = 1)$ and $((s_k = 0)$ in a period (k) with no shock on preferences
- (S_k > 1) and (s_k > 0) if a period (k) is more strongly valorized because of the depreciation of the future and of a higher un-stability. The consumer has then an incentive to consume less in the current period (k) and to realize precautionary saving.
- $(S_k < 1)$ and $(s_k < 0)$ in a period (k) which is less strongly valorized. Current consumption is then not delayed, because of a higher stability and of a stronger confidence in the future, and precautionary saving can be limited.

The recent worldwide COVID-19 health crisis can probably be classified as a recessionary situation increasing the preference for the current period ($s_t>0$) because of a fear of the future.

Besides, we suppose that the utility function of a representative household has the form:

$$U_t = \alpha_c \log(C_t) + \alpha_g \log(G_t) - \alpha_l \frac{1}{(1+\varphi)} L_t^{(1+\varphi)}$$
(12)

With, in period (t): (*C*): real consumption of private goods; (*G*): real public expenditure (consumption of public goods); (*L*): Labor supply.

The indices $(0 < \alpha_{s} < 1)$, $(0 < \alpha_{s} < 1)$ and $(0 < \alpha_{l} < 1)$ are the respective weights given by the representative consumer to consumption of private goods, public goods and leisure.

Utility is an increasing and concave function of (C), an index of the household's private consumption of all goods that are supplied, and of public goods and services provided by the government (G). Utility is also a decreasing and convex function of labor supply (L), where $(\phi>0)$ is the inverse of the Frisch elasticity of labor supply. Furthermore, this maximization is subject to an inter-temporal budgetary constraint. If we suppose complete financial markets, the flow budget constraint for each period (t) of the representative consumer is as follows:

$$(1+t^{c})P_{t}C_{t} + P_{t}INV_{t} + B_{t} = (1-t^{k})(R_{t}-\delta P_{t})K_{t} + (1-t^{l})W_{t}L_{t} + (1+R_{t-1})B_{t-1}$$
(13)

With, in period (*t*): (*B*_{*j*}): nominal value of government' bonds and public debt at the end of period (*t*); (*t*): labor taxation rate; (*t*): consumption taxation rate; (*t*^{*k*}): capital taxation rate.

Indeed, regarding his expenditure, the representative consumer consumes private goods, and he invests in capital or he purchases government' bonds. Regarding his resources, he receives labor (wage) and capital (interest rate) revenues. The representative consumer also receives gains from government bonds holding from the previous period. Besides, capital is not fully taxed: physical capital depreciation is exempted from taxation.

So, the maximization of equation (9) using (12) under the constraint (13) implies the following first order Euler conditions, regarding inter-temporal substitution, (?t), $(?n\geq 1)$:

$$C_t = \frac{E_t(P_{t+n})}{E_t(S_{t+n})\beta^n E_t[(1+R_{t+n-1})\dots(1+R_t)]P_t}E_t(C_{t+n})$$
(14)

So, in logarithms and in variations, with $[E_t(s_{t+1}) = \rho_s s_{j}]$, equation (14) implies:

$$c_t = E_t(c_{t+1}) - [R_t - E_t(p_{t+1})] - \rho_s s_t$$
(15)

So, private consumption increases with expected future consumption, and it decreases with the real interest rate. But private consumption also decreases with ($S_i > 1$), with a positive shock on preferences for the current period, and with (ρ_i), with the persistence of this shock. Fear about the future expected framework increases precautionary saving, and therefore, economic agents have an incentive to spend and to consume less today.

Regarding the substitutability between labor and leisure for the representative household, equations (9), (12) and (13) imply:

$$L_{t+n} = \left[\frac{\alpha_c (1 - t^l) W_{t+n} S_{t+n} \beta^n}{\alpha_l (1 + t^c) P_{t+n}}\right]^{\frac{1}{\varphi}} C_{t+n}^{-\frac{1}{\varphi}}$$
(16)

So, in logarithms and in variations, we obtain:

$$E_k(\mathbf{l}_{t+n}) = \frac{1}{\varphi}(w_{t+n} - p_{t+n}) + \frac{1}{\varphi}E_k(s_{t+n}) - \frac{1}{\varphi}c_{t+n}$$
(17)

As in traditional New-Keynesian models, labor supply increases with the real wage and decreases with private consumption. In the current modelling, labor supply also increases with a shock on households' preferences ($s_i > 0$), as a devaluation of the future can increase hours worked ('precautionary labor supply'). However, the decrease of the real wage can then more than compensate

for the previous effect. Our modelling is then compatible with the stylized fact mentioned by Basu and Bundwick (2017), i.e.: the empirical decrease (and not the increase) of the labor force employed in case of a shock on households' preferences (see section 3).

2.3. Budgetary and Monetary Policies

We suppose that capital is perfectly mobile, and therefore, the net-of-tax rate of return of the capital stock must equalize a real world-wide capital rate of return $(R^{m})^{2}$:

$$\frac{(1-t^k)(1+R_t)}{P_t} = (1+R^m)$$
(18)

In this context, equation (18) implies: $R_t = p_t$ (19)

Therefore, the central bank fixes the nominal interest rate in order to stabilize the real capital return in equation (19). In case of inflationary tensions, the nominal interest rate increases to conduct a contra-cyclical monetary policy and to avoid the out-burst of the economy, whereas in case of deflationary tensions, the nominal interest rate decreases to sustain economic growth.

Budgetary authorities fix national public expenditure; we suppose that all government debt is risk-free real debt (not state-contingent). The government is supposed to credibly commit to repay the public debt, and taxation rates are time-invariant. The budgetary constraint of the government is then as follows:

$$B_t = (1 + R_{t-1})B_{t-1} + P_t G_t - t^c P_t C_t - t^l W_t L_t - t^k (R_t - \delta P_t) K_t$$
(20)

The public debt in period (t) equals the public debt in the former period (t-1) increased by the former public debt's interest rate, plus the current period's public expenditure to be financed, decreased by fiscal resources. These fiscal resources include consumption, labor and capital taxation, considering that physical capital depreciation is exempted from taxation.

Besides, equations (3), (4) and (20) give the following ratio of nominal public debt in proportion of nominal GDP:

$$B_t^{Ra} = \frac{B_t}{P_t Y_t} = \frac{(1+R_{t-1})}{(1+p_t)(1+y_t)} \left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}}\right) + \frac{(G_t - t^c C_t)}{Y_t} - (1-\alpha)\frac{t^l}{\mu_t^L} - \frac{\alpha t^k}{\mu_t^K} \left(1 - \frac{\delta P_t}{R_t}\right)$$
(21)

If we log-linearize equation (21), using equation $(18)^3$, we obtain:

$$b_{t}^{Ra} = (1 + R - p - y)b_{t-1}^{Ra} + (R_{t-1} - p_{t} - y_{t}) + \frac{PG}{B}g_{t} - \frac{t^{c}PC}{B}c_{t} + \frac{P(t^{c}C - G)}{B}y_{t} + \frac{(1 - \alpha)t^{l}PY}{\mu^{L}B}\log(\mu_{t}^{L}) + \frac{\alpha t^{k}PY}{\mu^{K}B}\left(1 - \frac{\delta P}{R}\right)\log(\mu_{t}^{K})$$
(22)

We can normalize the equilibrium nominal price level (P=1), which implies ($P_t = 1 + p$). Thus, with (B=0) in the long run, equation (22) gives the following two equations:

$$b_t^{Ra} = (1 + R - p - y)b_{t-1}^{Ra} + (R_{t-1} - p_t - y_t)$$
(23)

$$\frac{G}{Y}g_{t} - t^{c}\frac{C}{Y}c_{t} + \frac{(t^{c}C - G)}{Y}y_{t} + \frac{(1 - \alpha)t^{l}}{\mu^{L}}\log(\mu_{t}^{L}) + \frac{\alpha t^{k}(R^{m} + t^{k} - \delta + \delta t^{k})}{\mu^{K}(R^{m} + t^{k})}\log(\mu_{t}^{K}) = 0$$
(24)

2.4. Equilibrium of the Model

We can now derive the equilibrium on the goods market regarding global demand. If we suppose that net exports are negligible, clearing the goods market requires the following:

$$Y_t = C_t + G_t + INV_t \tag{25}$$

Therefore, in logarithms and variations, equation (25) implies:

$$y_t = \frac{C}{Y}c_t + \frac{G}{Y}g_t + \frac{INV}{Y}inv_t$$
(26)

The constant long term capital-investment share (δ) implies ($INV_{\tau} \sim \delta K$). Therefore, equations (3) and (18) imply the following investment share in GDP:

$$\frac{INV_t}{Y_t} = \frac{INV_t}{K_t} \frac{K_t}{Y_t} \sim \frac{\delta\alpha (1 - t^k) P_t}{\mu_t^K [(1 + R^m) P_t - (1 - t^k)]}$$
(27)

The previous equations imply that the capital stock is constant in the long term $[|imv_i = k_i = 0]$. More precisely, private investment only replaces the capital stock from the previous period which is depreciated. In this framework, the global demand function of our model is⁴:

$$y_t = E_t(y_{t+1}) - \theta[R_t - E_t(p_{t+1}) - \overline{R_t}]$$
(28)

$$\theta = \frac{\mu^{K}(R^{m} + t^{k})\left[(1 + t^{c})\mu^{L}\frac{c}{\gamma} + (1 - \alpha)t^{l}\right]}{\left[\mu^{K}(R^{m} + t^{k})(1 + t^{c})\mu^{L}\frac{c}{\gamma} - \mu^{K}(R^{m} + t^{k})(\alpha + \varphi)t^{l} + \alpha\mu^{L}t^{k}(R^{m} + t^{k}) + \alpha\mu^{L}\delta(1 - t^{k})^{2}\right]}$$
$$\overline{R_{t}} = -\frac{\left[\rho_{s}(1 + t^{c})\mu^{L}\frac{c}{\gamma} + (1 - \alpha)t^{l}\right]}{\left[(1 + t^{c})\mu^{L}\frac{c}{\gamma} + (1 - \alpha)t^{l}\right]}s_{t} - \frac{(1 + \varphi)(1 - \alpha)(1 - \rho_{a})t^{l}}{\left[(1 + t^{c})\mu^{L}\frac{c}{\gamma} + (1 - \alpha)t^{l}\right]}a_{t}$$

- (θ): real interest rate elasticity of demand, inter-temporal elasticity of substitution
- $(\overline{R_t})$: Equilibrium or natural real interest rate, which corresponds to the steady-state real rate of return if prices and wages were fully flexible.

So, according to equation (28), higher future expected output increases current output and consumption, because households prefer to smooth consumption, and then higher future revenues raise their current consumption and current output. Current output is also a decreasing function of the excess of the real interest rate above its natural level, because of the inter-temporal substitution of consumption. Current output also increases with (-s), with a shock where current consumption is not delayed because of a stronger confidence in the future. Current output also increases with (-a), as labor demand then increases more than the negative technological shock in order to compensate for the weaker labor productivity.

Regarding the supply function of the model, the optimal strategy of the firm is to fix prices at nominal marginal production costs (*mc*). In a Calvo-type framework of staggered priced with monopolistically competitive firms, (0 < v < 1) is the fraction of goods prices which remain unchanged each period. We have then the following inflation rate:

$$p_t = \beta E_t(p_{t+1}) + k_1(mc_t - p_t) with k_1 = \frac{(1 - v)(1 - v\beta)}{v}$$
(29)

According to equations (3) and (4), the variable production cost of the quantity (Y) is:

$$CV_t = (R_t K_t + W_t L_t) = \left[\frac{\alpha}{\mu_t^K} + \frac{(1-\alpha)}{\mu_t^L}\right] P_t Y_t$$
(30)

Therefore, in logarithms and in variations from long run equilibrium values, we obtain the following variation of marginal production costs:

$$(mc_t - p_t) = -\frac{\alpha \mu^L}{[\alpha \mu^L + (1 - \alpha) \mu^K]} \log(\mu_t^K) - \frac{(1 - \alpha) \mu^K}{[\alpha \mu^L + (1 - \alpha) \mu^K]} \log(\mu_t^L)$$
(31)

So, by combining previous equations, the supply function is:

$$p_t = \beta E_t(p_{t+1}) + k_1 k_2 (y_t - y_t^p)$$
(32)

$$k_{2} = \frac{\mu^{L} \left\{ \alpha (1-\alpha) \left[\delta \left(1-t^{k} \right)^{2} + \left(t^{k} - t^{l} \right) \left(R^{m} + t^{k} \right) \right] + \left[\mu^{K} (1+\varphi) - \alpha \mu^{L} \right] (1+t^{c}) \left(R^{m} + t^{k} \right) \frac{c}{\gamma} \right\}}{\left[\alpha \mu^{L} + (1-\alpha) \mu^{K} \right] (R^{m} + t^{k}) \left[(1+t^{c}) \mu^{L} \frac{c}{\gamma} + (1-\alpha) t^{l} \right]}$$

$$y_t^p = \frac{(1-\alpha)\mu^K (1+t^c)\mu^L \frac{\sigma}{Y}}{k_2 [\alpha \mu^L + (1-\alpha)\mu^K] \left[(1+t^c)\mu^L \frac{\sigma}{Y} + (1-\alpha)t^l \right]} [(1+\varphi)a_t + s_t]$$

Therefore, in our model, prices increase with the excess of economic activity above its potential level, which would prevail in absence of price rigidities. Potential output increases with a positive technological shock improving labor productivity (a_i) , and it also increases with (s_i) , if the consumer has a higher incentive to generate more precautionary saving today.

2.5. Calibration

We will consider a standard calibration of the parameters of our model. However, in the rest of the paper, we will make a detailed sensitivity analysis, in order to study the influence of a variation of the parameters of our model on our theoretical results. Regarding the long term and equilibrium relative shares of the various components of global demand in GDP, according to average values in the European Union for example, we consider that the share of private consumption in GDP is $(\frac{C}{\gamma} = 0.55)$, which implies a share of public consumption in GDP: $(\frac{INV}{\gamma} = 0.22)$. We consider that the consumption taxation rate is ($t^{e} = 0.3$), the capital taxation rate is ($t^{e} = 0.25$), whereas the labor taxation rate is ($t^{e} = 0.22$). We make the hypothesis that the long term growth rate is (y = 1.8%).

Regarding households, we suppose that the time discount factor is ($\beta = 0.99$), whereas the inverse of the Frisch elasticity of labor supply is ($\varphi=2$). We make the hypothesis that the persistence of the shock on households' preferences is ($\rho_s = 0.18$), whereas its variance is($\sigma_t^{s2} = 0.0075$). Regarding firms, we suppose that the capital depreciation rate is ($\delta = 0.07$). The output elasticities are

respectively (α =0.34) for capital and (1- α =0.66) for labor. We make the hypothesis that the persistence of the productivity shock is ($\rho_a = 0.95$), whereas its variance is($\sigma_t^{a2} = 0.004$). The fraction of goods prices which remains unchanged is ($\nu = 0.75$). The world-wide capital return is (R^m =-0.18)⁵. Finally, we make the hypothesis that (μ^{K} = 1.18) and (μ^{L} = 1.32) are the long-run markups over capital and labor marginal costs.

3. CONSEQUENCES OF SHOCKS ON PREFERENCES OR ON PRODUCTIVITY

We can now study and compare the consequences of two different uncertainty shocks on main economic variables: demand factors and prices. Our simulations show that a demand shock on the preferences of the representative household or a productivity shock are recessionary on all parameters of global demand. However, the labor force seems to decrease in case of a shock on households' preferences but to increase in case of a negative labor productivity shock. Besides, prices variables are very differently affected in case of each specific shock.

First, we can mention that in the framework of our model, the long term economic growth rate (y) has no consequences on current demand or prices global economic variables, and only affects the public debt level. Our sensitivity analysis also shows that the influence of the capital taxation rate (t^k) is very limited; indeed, according to equation (18), the real capital return is largely fixed according to a worldwide rate of return (R^m) . However, we can analyze the influence of other structural parameters of our model on demand and prices variables in case of demand (on households' preferences) or supply (on labor productivity) shocks.

3.1. Consequences on Demand Factors of a Shock on Households' Preferences

In this section, we study the consequences of a demand shock on the preferences of the representative household ($s_t > 0$). When the global context is more uncertain (like for example with the COVID-19 crisis), this shock gives an incentive to consume less in the current period, in order to realize precautionary saving for an unstable future economic situation. Then, according to the basic calibration of our model, the decrease of private consumption remains limited ($\frac{\partial c_t}{\partial s_t} = -0.08$). However, the decrease of global economic activity ($\frac{\partial y_t}{\partial s_t} = -0.32$), and mainly the decrease of public

expenditure $(\frac{\partial g_t}{\partial s_t} = -1.18)$ are much more accentuated. In this context, what are the results of the sensitivity analysis of our various parameters?

First, decreases of economic activity and of labor demand are accentuated if the share of private consumption in GDP (C/Y) is weak, whereas the decrease of public consumption is then more moderate (see Figure 1); the decrease of private consumption is then insignificantly weaker. Indeed, a demand shock on the preferences of the representative consumer ($s_i > 0$) increases the incentive to consume today, because of a higher degree of uncertainty about the future. Therefore, if the share of private consumption in GDP is weak, current private consumption remains more weakly negative than other demand factors, but its weak share isn't sufficient to sustain global demand. Recession and the large decrease of current economic activity cannot be avoided, as uncertainty also depresses forecasts about future economic activity.

Besides, the influence of the capital taxation rate (t^{k}) on economic variables is very moderate, provided this rate is sufficiently high ($t^{k} > -R^{m} = 0.18$ with our basic calibration). Indeed, according to equation (18), capital profitability largely depends on foreign economic variables. However, global economic recession and the decrease of public expenditure seem accentuated if the labor taxation rate (t^{k}) is high, or if the consumption taxation rate (t^{k}) is weak, even if private consumption is then slightly less depressed (see Figure 1). Therefore, in case of a demand shock on the preferences of the representative consumer, increasing the weight on consumption taxation, and reducing the weight on labor taxation, would allow to decrease global recessionary tensions and to limit the collapse of global demand.

On the labor market, in case of a shock on households' preferences $(s_t > 0)$, the capital stock is stabilized whereas labor demand by firms decreases $(\frac{\partial l_t}{\partial s_t} = -0.49$ with our basic calibration). Indeed, labor demand is reduced proportionately to the recessionary effect of the shock $[l_t = \frac{1}{(1-\alpha)}y_t]$. So, the decrease of the labor force is more moderate if the recession is more limited, i.e. if the share of private consumption in GDP (C/Y) is high, if the labor taxation rate (t) is relatively weak, whereas the consumption taxation rate (t) is high (see Figure 1).

In this framework, regarding empirical data, in 2021, the statutory personal income tax rate depending on the threshold of revenues is probably higher in Italy, in Belgium, in the Netherlands or in the United-Kingdom, which could

have contributed to increase the economic consequences of the COVID-19 shock in these countries. On the contrary, labor taxation rates were weaker in Switzerland, in Finland, in Luxembourg or in Norway, which could have helped to lighten the weight of the crisis in these countries. Furthermore, in 2021, standard VAT rates were around 20% in OECD countries, but they were higher in Hungary (27%), in Denmark, in Sweden or in Norway (25%), where the strength of the crisis was more limited.





Regarding the parameters of the representative firm, in case of a shock on households' preferences $(s_t > 0)$, the recession appears as significantly more limited if the capital share in the production function (α) is high. Indeed, a higher capitalization implies that a same decrease of labor demand is compatible with a weaker recession $[l_t = \frac{1}{(1-\alpha)}y_t]$. The decrease of public expenditure is

then consistently limited, even if the decrease of private consumption is slightly accentuated (see Figure 2). Besides, the recession is also less accentuated if the long run markup over labor marginal costs (μ^L) is higher. Indeed, for firms, a higher expected profitability of the labor production factor reduces the decrease of labor demand and of economic activity, even if this higher markup is detrimental to the purchasing power of households and slightly accentuates the decrease of private consumption.

In the framework of our model, the recession is also more limited if prices are more flexible (v is weak). Indeed, the recessionary framework is then accompanied and self-sustained by a strong deflationary framework regarding prices [see equation (32)]. As the real wage is less reduced, this is beneficial to private consumption, which can even increase if (v) is below a given value [v<0.66 with our basic calibration]. The employed labor force, as well as mainly public expenditure, are then also less reduced (see Figure 2).





Regarding households' parameters, the recession is slightly more limited if the Frisch elasticity of labor supply is high (φ is weak). Indeed, if this parameter is high, the markup over labor marginal costs (μ^L) is weaker, and the decrease of wages is reduced, in order to avoid salary costs detrimental to employment and to labor supply. As the decrease of the real wage is more limited, the decrease of the labor force and of global economic activity are weaker, and the decrease of public expenditure is mainly strongly attenuated (see Figure 3).

Besides, obviously, the recession is accentuated if the persistence of the shock on the preferences of households (ρ) is high. Indeed, the recessionary

consequences of the shock on preferences downgrading more uncertain future periods are then more long lasting, and weaker future expectations have then lasting recessionary consequences on economic variables (see Figure 3). In the same way, the recession is also slightly accentuated if the discount rate of the future ($\rho_{,j}$) is high. Indeed, the demand shock on preferences is relatively persistent, and it can then spread recessionary consequences on many future periods. The decrease of the real wage is then slightly accentuated, while the markup over labor marginal costs slightly increases.





3.2. Consequences on prices factors of a shock on households' preferences

In case of a recessionary shock on households' preferences $(s_t > 0)$, prices decrease $(\frac{\partial p_t}{\partial s_t} = \frac{\partial R_t}{\partial s_t} = -0.19$ with our basic calibration), and the monetary policy of the central bank is to reduce proportionately the nominal interest rate, in order to limit these deflationary tensions. The nominal $(\frac{\partial w_t}{\partial s_t} = -2.25)$ and the real wage $(\frac{\partial (w_t - p_t)}{\partial s_t} = -2.06)$ also decrease, in a context of weaker economic activity and labor demand. Therefore, because of this weaker real wage, the markup over labor marginal costs increases $(\frac{\partial \log (\mu_t^L)}{\partial s_t} = 2.22)$ [see equation (4)], whereas the markup over capital marginal costs decreases $(\frac{\partial \log (\mu_t^K)}{\partial s_t} = -0.32)$ at

the same pace as global economic activity. More precisely, what are the results of our sensitivity analysis?

Prices and wages decrease less, and therefore, the markup over labor marginal costs is more limited, whereas the markup over capital marginal costs is less reduced, if the recession is weaker, i.e. if the share of private consumption in GDP (C/Y) is high, if the weight of labor taxation (t) is weak, whereas the weight of consumption taxation (t) is high. In the same way, prices and real wages decrease less, and therefore, the markup over labor marginal costs is more limited whereas the markup over capital marginal costs is less reduced, when the recession is more limited, i.e. if the persistence of the shock on households' preferences (p) is weak, or if the long term and equilibrium markup over labor marginal costs (μ ^L) is high. The recession is also much more limited if prices are more flexible (v is weak). Indeed, real wages then decrease less, as the deflation is accentuated by prices flexibility.

Finally, regarding public indebtedness, the public debt to GDP ratio slightly increases $(\frac{\partial b_t}{\partial s_t} = 6.6\%$ with our basic calibration) with a recessionary shock on households' preferences. However, our modelling also allows to underline the ambiguity of price flexibility. Indeed, we have seen that the latter and a more deflationary framework limit the global recession and the decrease of public expenditure. However, the public debt to GDP ratio is mainly much more limited if prices rigidity (v) is high; the public debt can then even decrease. Indeed, if prices are too flexible, the recession could imply an excessively deflationary situation, harmful to the containment of the nominal public debt to GDP ratio.

Furthermore, as mentioned by Basu and Bundick (2017), uncertainty per se has a very limited influence to increase the variance of economic activity and of other economic variables in case of a shock on households' preferences. Indeed, uncertainty per se slightly increases the recession the first year after the shock; but the effect then quickly vanishes afterwards (see Figure 4). An extremely high variance $[V(s_t)=0.05]$ of this shock on households' preferences would be necessary to imply a potential recession of $[\partial y_t = -0.42\partial s_t]$ and a potential deflation of $[\partial \pi_t = -0.34\partial s_t]$ the first year after the shock. In fact, the main consequence of a higher uncertainty would be to accentuate the potential variation of the public debt, which could either increase or decrease according to uncertainty about the shock on households' preferences.



Figure 4: Consequences of uncertainty on a shock on households' preferences

3.3. Consequences on demand factors of a productivity shock

A negative supply and technological shock $(a_i < 0)$ decreases labor productivity. Therefore, because of this weaker efficiency of the labor factor, labor demand must increase $(\frac{\partial l_t}{\partial (-a_t)} = 0.25$ with our basic calibration) in order to produce the same level of goods and services, whereas private consumption decreases $(\frac{\partial c_t}{\partial (-a_t)} = -0.74)$ with the weaker purchasing power of households. Besides, the weaker productivity of the private sector implies a recession and a decrease of global economic activity and demand $(\frac{\partial y_t}{\partial (-a_t)} = -0.49)$, whereas public expenditure also decreases $(\frac{\partial g_t}{\partial (-a_t)} = -0.39$. In this context, what are the results of the sensitivity analysis of our various parameters?

First, in case of a negative productivity shock, the decrease of private consumption is reduced and the increase of labor demand is more limited if the share of private consumption in GDP (C/Y) is high. Nevertheless, the recession is then slightly accentuated (see Figure 5). Indeed, a negative shock on labor productivity decreases the real wage and the purchasing power of households, which reduces private consumption for the current and future periods. Therefore, this decrease of private consumption is more strongly limited if the share of private consumption in GDP is high. On the labor market, in case of a negative productivity shock ($a_t < 0$), labor demand by firms slightly increases to compensate for the weaker productivity of this production factor, and to produce the same level of goods and services $[l_t = \frac{1}{(1-\alpha)}y_t - a_t]$. However, the increase of labor demand is more moderate if the share of private consumption in GDP (C/Y) is high, as the recession is then slightly accentuated.

Furthermore, according to our model, taxation rates have only a very limited influence on global economic activity and labor demand. Indeed, if the labor taxation rate (t) is weak or if the consumption taxation rate (t) is high, the recession is very slightly accentuated. The decrease of private consumption is then more reduced, whereas the decrease of public expenditure is higher (see Figure 5), as the necessity to replace a weak private demand is limited.

Figure 5: Variation of demand factors according to the share of private consumption in GDP or to the labor taxation rate



Regarding the parameters of the representative firm, the recession, the decrease of private and public consumption are more limited if the capital share in the production function (α) is high (see Figure 6). Indeed, the higher the capitalization of the economy, and the weaker the share of the labor force in the production function, the weaker the recessionary effect of the loss of productivity of this factor, which volume is more negligible. A weaker capital depreciation rate (δ) also reduces the decrease of private consumption and the increase of labor demand.

More significantly, in case of a negative labor productivity shock, economic growth and public expenditure are less reduced, and can even increase, if prices are more rigid (v is high). Indeed, because of the strong increase of the real wage, the decrease of private consumption is weaker if prices are more rigid; global economic activity then increases (see Figure 6). On the contrary, price flexibility and a deflationary framework imply a stronger recession.

Regarding households' parameters, global economic activity, private and public consumption are less reduced, whereas labor demand increases more, if the Frisch elasticity of labor supply is high (ϕ is weak). Indeed, if this parameter



Figure 6: Variation of demand factors according to the capital share in the production function or to prices rigidity

is high, the increase of the real wage has a stronger effect to increase also labor supply, the purchasing power of households, and to sustain private consumption. Besides, obviously, the recession is accentuated if the persistence of the negative productivity shock (ρ_{a}) is high. Economic activity, private and public consumption are more reduced, while labor demand increases less, if the degradation of labor productivity is more long lasting, as weaker future expectations have then lasting recessionary consequences. On the contrary, economic growth, private and public consumption are less depressed and labor demand increases more if the discount rate of the future (β) is weak.

3.4. Consequences on prices factors of a productivity shock

In case of a negative shock on labor productivity $(a_t < 0)$, the nominal $(\frac{\partial w_t}{\partial (-a_t)} = -0.11$ with our basic calibration) and the real wage $(\frac{\partial (w_t - p_t)}{\partial (-a_t)} = -0.23)$ decrease, in a context of weaker labor productivity. Then, because of this negative technological shock, the markup over labor marginal costs decreases $(\frac{\partial \log(\mu_t^L)}{\partial (-a_t)} = -0.51)$, whereas the markup over capital marginal costs also decreases $(\frac{\partial \log(\mu_t^K)}{\partial (-a_t)} = -0.49)$ at the same pace as global economic activity. There are also moderate inflationary tensions $(\frac{\partial p_t}{\partial (-a_t)} = 0.12)$.

However, inflationary tensions are slightly higher, wages decrease more, and thus the markup over labor marginal costs can be less deteriorated, if the share of private consumption in GDP (C/Y) is high, or if the consumption taxation rate (t) is high. Besides, if the labor taxation rate (t) is high, wages decrease less, the markups over labor and capital marginal costs are less deteriorated, whereas inflationary tensions are slightly accentuated.

In the same way, with a weaker productivity of the labor production factor, a higher capitalization and capital share in the production function (α is high) attenuates the decrease of wages, of markups over labor and capital marginal costs, as well as inflationary tensions. If the capital depreciation rate (δ) is high, the decrease of the markup over labor marginal costs is accentuated; however, the decrease of wages and of the markup over capital marginal costs are slightly limited. If the discount rate of the future (β) is weak, the markup over capital marginal costs is slightly reduced, but wages are higher and could increase; then, the decrease of the markup over labor marginal costs is strongly accentuated.

Furthermore, real wages increase less and they can even decrease, whereas the decrease of the markup over labor marginal costs is more limited, if the persistence of the productivity shock (ρ^a) is high; inflationary tensions are then also more limited. In a context of price rigidity (v is high), the strong increase of wages implies a strong decrease of the markup over labor marginal costs. However, deflation is then avoided, prices slightly increase, and the decrease of the markup over capital marginal costs is slightly reduced. On the contrary, in a context of price flexibility (v is weak), deflation implies that a negative productivity shock is more recessionary. Real wages are then reduced and they can slightly decrease, whereas the decrease of the markup over labor marginal costs is strongly limited; the latter can even increase.

Regarding public indebtedness, the public debt to GDP ratio decreases $(\frac{\partial b_t}{\partial (-a_t)} = -4.57$ with our basic calibration) with a negative shock on labor productivity. Indeed, inflationary tensions contribute to decrease the public indebtedness level.

Finally, as mentioned by Basu and Bundick (2017), uncertainty per se has a limited influence to increase the variance of economic activity and of other economic variables in case of a negative productivity shock which would be long lasting. Indeed, uncertainty per se increases the recession the first year after the shock (see Figure 7). An extremely high variance [V(a)=0.05] of this negative productivity shock would be necessary to imply a potential recession of $[\partial y_t = -0.56\partial a_t]$ or a potential inflation of $[\partial \pi_t = 0.19\partial a_t]$ the first year after the shock. In fact, the main consequence of a higher uncertainty would be

to accentuate the potential variation of the public debt, which could decrease by between $[\partial b_t = -5.24\partial a_t]$ and $[\partial b_t = -3.89\partial a_t]$ according to the intensity of the negative productivity shock.



Figure 7: Consequences of uncertainty on a negative productivity shock

CONCLUSION

We have studied the consequences on various economic and prices variables of an uncertainty shock, of a shock on the preferences of households (demand shock) or on labor productivity (supply shock). The main results of our model are then as follows. A demand shock on the preferences of the representative household increases uncertainty about the future and the incentive to create precautionary saving. Therefore, this deteriorates anticipations for all future periods, and it also creates recessionary tensions for the current period. In this context, the decrease of global economic activity and of labor demand are more limited if the relative weight of private consumption in GDP or on consumption taxation are high, whereas the relative weight on labor taxation is weak. Besides, the recession appears also as more limited in countries with high prices flexibility, with a high capital share in the production function, or with a high Frisch elasticity of labor supply. In case of such a negative demand shock on preferences, prices (deflation), real and nominal wages decrease, whereas the markup over labor marginal costs increases, and the markup over capital marginal costs decreases.

A negative supply shock on labor productivity is also recessionary on global economic activity: it implies a decrease of public expenditure and of private consumption, whereas a higher volume of labor force must be employed to compensate for its weaker productivity. Besides, in case of a negative productivity shock, the nominal and the real wage decrease; then, the markups over labor and capital marginal costs both decrease, in a context of moderate inflationary tensions. Nevertheless, the decrease of private consumption and the increase of labor demand remain more moderate if the share of private consumption in GDP or if the weight on consumption taxation are high, whereas the weight on labor taxation is weak. Furthermore, after a negative productivity shock, the recession is more limited if the capital share in the production function is high. Regarding the consequences of uncertainty per se, the variance of a shock on households' preferences or of a negative productivity shock has limited consequences on global economic variables. The main consequence of uncertainty would be to increase the variance of the public debt, and to limit the capacity of the government to control the public debt level.

Notes

- 2. With (R > 0), equation (18) implies: $t^k = \frac{(R-R^m)}{(1+R)} > -R^m$.
- 3. Equation (18) implies: $\frac{P_t}{R_t} = \frac{(1-t^k)P_t}{[(1+R^m)P_t-(1-t^k)]}$, and $\log\left(1-\frac{\delta P_t}{R_t}\right)$ is negligible.
- 4. The precise derivation of the analytical results of our model are available upon request.

5. With our basic calibration, this value is compatible with a nominal interest rate: $R = \frac{(R^m + t^k)}{(1 - t^k)} = 9.33\%$, a capital output ratio: $\frac{K}{Y} = \frac{\alpha}{\mu^K R} = 3.09$, and an investment share in GDP: $\frac{INV}{Y} = \frac{\delta \alpha (1 - t^k)}{\mu^K (R^m + t^k)} = 21.61\%$.

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